Instructions: Suppose your student number is

20XY-A**BCD**E

with binary representation of BCD given by

$$(\text{BCD})_{10} = (\text{n}_1\,\text{n}_2\,\text{n}_3\,\text{n}_4\,\text{n}_5\,\text{n}_6\,\text{n}_7\,\text{n}_8\,\text{n}_9\,\text{n}_{10}\,)_2\,.$$

For each $i \in \{1, 2, 3, ..., 10\}$, if $\text{n}_i = 0$, answer item i(a), else if $\text{n}_i = 1$, answer item i(b) instead.

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e.g. if the middle three digits of your student number is 072 with binary representation

$$(072)_{10} = (0\,0\,0\,1\,0\,0\,1\,0\,0\,0)_2\,,$$

you need to answer 1(a), 2(a), 3(a), 4(b), 5(a), 6(a), 7(b), 8(a), 9(a), and 10(a).

Work Independently! Do not consult anyone except your instructor about these problems.

Questions:

I. Evaluate the following integrals *(4 pts each)*

1. Integrals of Powers

   (a) $\displaystyle\int \left(\sqrt{x} - 1\right)^6 \, dx$

   (b) $\displaystyle\int \frac{z^2 - 1}{\left(z^2 + z + 1\right)^2} \, dz$

2. Integrals of Trigonometric Functions

   (a) $\displaystyle\int \sqrt{\frac{\sin y}{\cos^5 y}} \, dy$

   (b) $\displaystyle\int \left(\sec^2\theta - \cos^2\theta\right) \tan\theta \, d\theta$

3. Definite Integral

   (a) $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos\beta \cos\left(\pi \sin\beta\right) \, d\beta$

   (b) $\displaystyle\int_1^4 \frac{1}{\sqrt{w}\left(\sqrt{w} + 2\right)^3} \, dw$

4. Integrals involving Absolute Value

   (a) $\displaystyle\int_0^{\frac{\pi}{2}} \left|\cos u - \frac{1}{2}\right| \, du$

   (b) $\displaystyle\int_0 \left|3s^2 - 2s - 1\right| \, ds$

II. Do as indicated.

5. Absolute Extrema *(4 pts)*

   (a) Find the absolute extrema of $f(x) = -4x^3 - 9x^2 + 12x - 3$ on the interval $[0, 1]$.

   (b) Find the absolute extrema of $f(x) = 4x^3 - 9x^2 - 12x - 3$ on the interval $[-1, 0]$.

6. Optimization *(5 pts)*

   (a) An open box is to be made from a 10-cm by 10-cm piece of cardboard by cutting out squares of equal size from the four corners and bending up the sides. What size should the squares be to obtain a box with the largest volume?

   (b) Find the least amount of material that can be used to construct a rectangular box with an open top and square base if its volume is 32 in^3.

7. Average Value *(2 pts)*

   (a) Find $b > 0$ such that the average value of $g(x) = 6x^2$ on $[0, b]$ is equal to 16.

   (b) If g is continuous on $[1, 3]$ and $\displaystyle\int_1^3 g(x)\, dx = 4$, show that there is a $c \in [1, 3]$ such that $g(c) = 2$.

8. Derivative of Integrals *(1 pt, 3 pts)*

(a) Let $H(x) = \displaystyle\int_{x}^{2x \sin \frac{x}{3}} \sqrt{2 + \left(\frac{t}{\pi}\right)^2}\, dt.$

i. Find $H\left(\frac{\pi}{2}\right)$

ii. Find $H'\left(\frac{\pi}{2}\right)$

(b) Let $H(v) = \displaystyle\int_{\tan x}^{x^3+x} \frac{2\cos v}{v+1}\, dv.$

i. Find $H(0)$

ii. Find $H'(0)$

9. Rectilinear Motion *(2 pts, 3 pts)*

(a) A ball is thrown vertically upwards with initial velocity of 32 ft/s from the top of a building. The ball hit the ground after 3 seconds. (Assume acceleration due to gravity is equal to -32 ft/s^2)

i. When will the ball reach its maximum height?

ii. What is the height of the building?

(b) The acceleration, in m/sec^2, of a particle moving along a line at t seconds is given by $a(t) = 12$. If at $t = 1$, the particle is moving at the speed of 6 m/sec and is one unit to the right of the origin.

i. Find the velocity of the particle when $t = 2$.

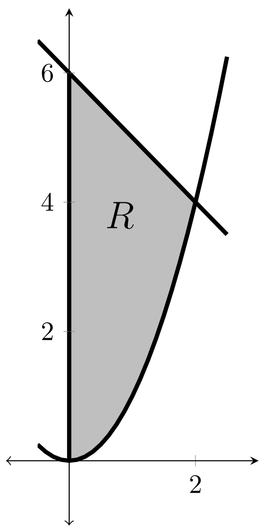
ii. Find the position of the particle when $t = 0$.

III. For the following regions R, **SET UP** the integral(s) needed to find the following: *(3 pts each)*

1. the area of region R
2. the perimeter of region R
3. volume of the solid of revolution when R is revolved about the *given* line:
   (a) using the method of **Washers**
   (b) using the method of **Cylindrical Shells**

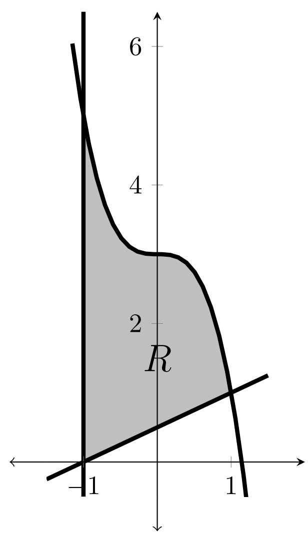
10. Plane Regions

(a) R bounded by $C_1 : y = x^2$, $C_2 : y = 6 - x$ and the y-axis.



Axis of revolution: $y = -1$

(b) R bounded by $C_1 : y = 3 - 2x^3$, $C_2 : y = \dfrac{x+1}{2}$ and the line $x = -1$.



Axis of revolution: $x = -2$

Total: 40 points (Bonus: 8 points) **papilo2017**